

# Helpman/Melitz/Yeaple (2004) : Export vs. FDI

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## 1 Introduction

Assumptions:

- $N$  countries indexed by  $i$
- Labor is the only production factor
- In each country, sector 1 produces a homogeneous product with one unit of labor per one unit of output. Assuming that the homogeneous good is freely traded, the wage rate equals 1 in every country. A fraction of  $1 - \sum_h \beta_h$  of income is spent on the homogeneous good
- $H$  sectors (indexed by  $h$ ) produce differentiated goods with a fraction  $\beta_h$  being spent on goods from any particular sector
- Firms in each sector  $h$  engage in monopolistic competition

Market entry:

- Each entrant pays a fixed cost of entry  $f_E > 0$  and then draws a labor-per-unit-output coefficient  $a$  (inverse of productivity) from a distribution  $G(a)$  (the PDF and CDF of the distribution will be denoted  $f_a$  and  $F_a$ , respectively).
- If an entrant chooses to start production with a given  $a$ , additional fixed costs  $f_D > 0$  have to be paid.
- If  $a$  is sufficiently low, the entering firm may additionally choose to export to foreign markets and has to pay an additional fixed cost  $f_X > 0$  for every export destination.
- If, on the other hand, a firm chooses to establish a foreign subsidiary, it has to pay fixed costs  $f_I > 0$ .
- It is assumed that a firm either exports or has foreign subsidiaries, but not both. This actually follows from the model as, depending on the productivity, either exports or subsidiaries are more profitable.
- Exports from  $i$  to  $j$  are subject to melting iceberg trade costs  $\tau_{i,j} > 1$ .

## 2 Preferences and demand

Preferences are modelled similarly to the standard Dixit/Stiglitz approach, however in this paper utility is a logarithmic function of the standard CES utility. The utility function is given as

$$U = \left(1 - \sum_{h=1}^H \beta_h\right) \log z + \sum_{h=1}^H \frac{\beta_h}{\alpha_h} \log \left( \int_{v \in V_h} x_h(v)^{\alpha_h} dv \right)$$

(see the working paper) where  $\alpha$  is the usual love-of-variety parameter with the elasticity of substitution being  $\epsilon = 1/(1 - \alpha) > 1$ . As the utility function is additively-separable with known income shares spent on each sector (see assumptions), it can be maximized for each sector separately (sector subscripts  $h$  will be dropped from now on):

$$\mathcal{L} = \frac{\beta}{\alpha} \log \left( \int x(v)^\alpha dv \right) - \lambda \left( \int p(v) x(v) dv - \beta E \right)$$

The resulting demand function takes the well-known form  $x = \text{const. } p^{-\epsilon}$  with constant demand elasticity:

$$x(v) = \frac{\beta E}{\int p(z)^{\alpha/(\alpha-1)} dz} p(v)^{1/(\alpha-1)} = \frac{\beta E}{\int p(z)^{1-\epsilon} dz} p(v)^{-\epsilon}$$

where  $E$  is the consumer's expenditure on all products. Note that since utility is logarithmic, it is not equivalent to real income using the standard Dixit/Stiglitz price index.

## 3 Firms

Due to monopolistic competition, firms set the product price as a mark-up over marginal costs. This mark-up is determined by the substitution parameter  $\alpha$  (as in the standard Dixit/Stiglitz model), while marginal costs depend on the firm-specific labor input coefficient  $a$  and the wage rate which is normalized at 1. Therefore, for domestically sold products,

$$p = \frac{a}{\alpha}$$

Due to mill pricing (ex works prices are the same for all products), the price for an exported good is

$$p = \frac{\tau_{i,j} a}{\alpha}$$

## 4 Equilibrium

From the standard Dixit/Stiglitz model profits can easily be determined as  $\pi = r/\epsilon - f$ ,  $r$  being the revenue and  $f$  fixed costs. In this model we have to differentiate whether a firm produces for the domestic market only, or either exports or has foreign subsidiaries in addition. In any case, a firm which exports or does FDI will produce for the domestic market as well (as the entry cost  $f_E$  has already been paid and productivity of firms which engage in exports/FDI is sufficiently high to yield positive profits in the domestic market). The profits from domestic production ( $\pi_{D,i}$ ), exports ( $\pi_{X,i,j}$ ) and FDI ( $\pi_{I,i}$ ) for home country  $i$  and export/FDI destination country  $j$  are given as

$$\pi_{D,i} = \frac{A_i p^{1-\epsilon}}{\epsilon} - f_D = \frac{A_i p^{1-\epsilon}}{\epsilon} - f_D = a^{1-\epsilon} B_i - f_D \quad (1)$$

$$\pi_{X,i} = (\tau_{i,j} a)^{1-\epsilon} B_j - f_X \quad (2)$$

$$\pi_{I,i} = a^{1-\epsilon} B_j - f_I \quad (3)$$

with  $B_i = (1 - \alpha) A_i / a^{1-\epsilon}$ , which can be seen as a measure for the sector-specific demand level.

Assuming  $0 < f_D < f_X \tau^{\epsilon-1} < f_I$  and  $\tau > 1$ , this can be plotted as a function of the labor input coefficient  $a$  or of  $a^{1-\epsilon}$ , which increases monotonically with productivity.

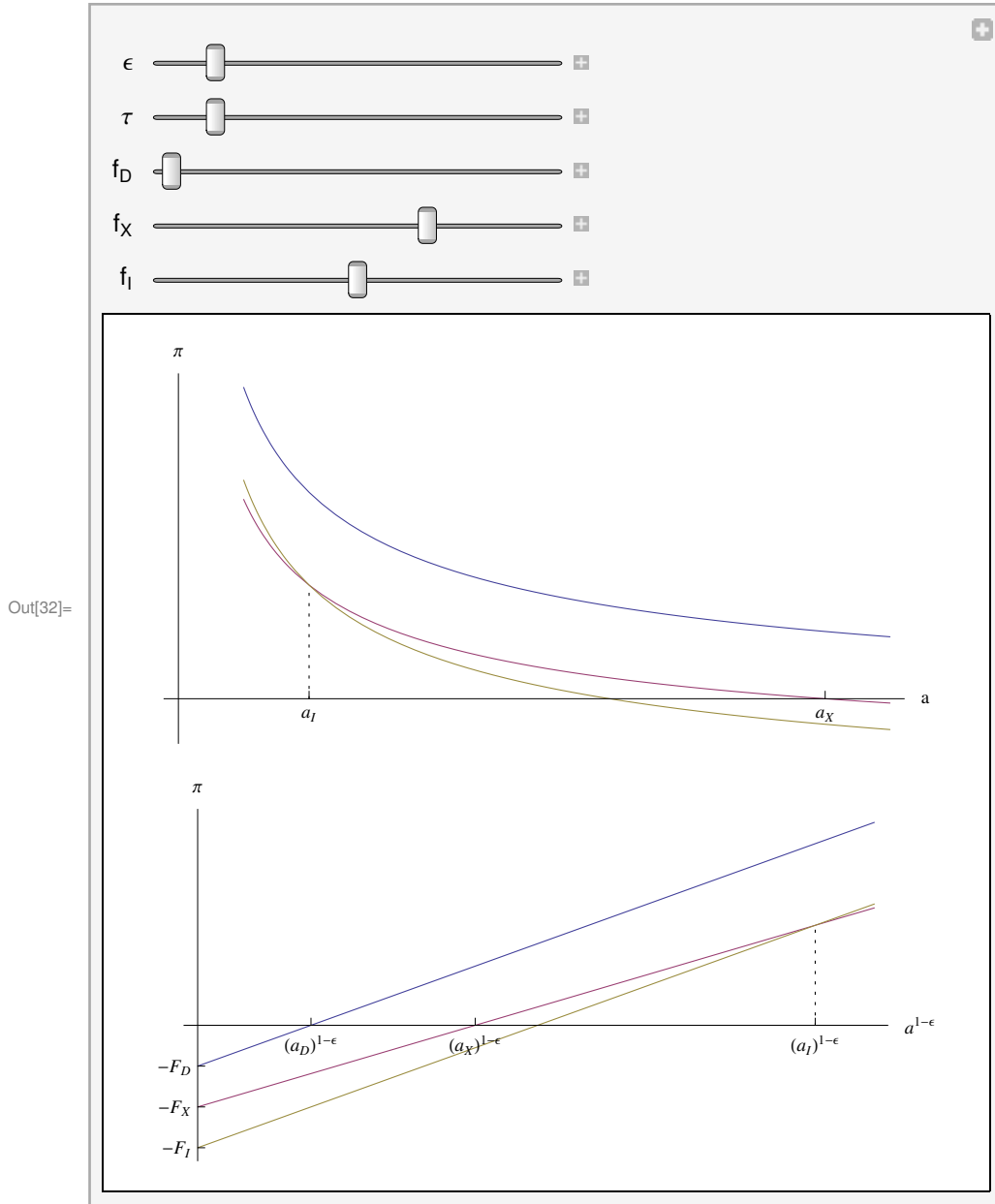
*The Mathematica code relevant for this paper is almost entirely in the package **HelpmanMelitzYeaple2004**, which is provided by the file **Helpman-MelitzYeaple2004.m**.*

*The package must be first installed using File → Install or manually copied to the folder \$UserBaseDirectory/Applications.*

```
In[30]:= << HelpmanMelitzYeaple2004'
```

```
In[31]:= allAssump := 1 > alpha > 0 && epsilon > 1 && tau > 1 && fI > fX tau^{epsilon-1} > fD > 0 && fE > 0 &&
n in Integers && n > 1 && B > 0;
```

In[32]:= manipulateCutoffLevels



The graph shows that firms with input coefficient  $a > a_D$  do not produce at all as they cannot earn a profit which at least covers fixed costs  $f_D$ . Firms with  $a_X < a \leq a_D$  produce for the domestic market only because they would not be able to pay for the fixed export or FDI costs. For firms with  $a_I < a \leq a_X$  it is profitable to export and produce for the domestic market, while firms with  $a \leq a_I$  choose FDI over exports.

### 4.1 Cutoff productivity levels

The cutoff input coefficient levels in equilibrium can be determined from the following equations:

$$a_{D,i} = \left( \frac{f_D}{B_i} \right)^{\frac{1}{1-\epsilon}} \quad (4)$$

$$a_{X,i,j} = \frac{(f_X/B_j)^{\frac{1}{1-\epsilon}}}{\tau_{i,j}} \quad (5)$$

$$a_{I,i,j} = \left( \frac{f_I - f_X}{B_j(1 - \tau^{1-\epsilon})} \right)^{\frac{1}{1-\epsilon}} \quad (6)$$

From these equations it is evident that  $a_D$  is decreasing in  $f_D$ ,  $a_X$  is decreasing in  $f_X$  and  $\tau$ , and  $a_I$  is increasing in  $f_X$  and  $\tau$ , while it decreases in  $f_I$ . Intuitively, higher fixed costs of an activity (domestic production, exports...) require higher productivity to break even and thus a lower  $a$ . Additionally, factors making exports less attractive (high fixed and variable export costs), lower the productivity level required for FDI to yield higher profits than exporting.

## 4.2 Relative market shares

These relationships can be used to derive effects on the relative market share of exporters and foreign subsidiaries in any country. From Eq. (2) and Eq. (3) the revenues of exporters from country  $i$  in country  $j$  and the revenues of affiliates in country  $j$  of country  $i$ 's firms are (aggregated over all productivity levels):

$$\begin{aligned} r_{X,i,j} &= \int_{a_{I,i,j}}^{a_{X,i,j}} (\tau_{i,j} a)^{1-\epsilon} B_j dF_a(a) \\ &= \int_0^{a_{X,i,j}} (\tau_{i,j} a)^{1-\epsilon} B_j dF_a(a) - \int_0^{a_{I,i,j}} (\tau_{i,j} a)^{1-\epsilon} B_j dF_a(a) \\ &= \tau_{i,j}^{1-\epsilon} B_j (V(a_{X,i,j}) - V(a_{I,i,j})) \\ r_{I,i,j} &= \int_0^{a_{I,i,j}} a^{1-\epsilon} B_j dF_a(a) = B_j V(a_{I,i,j}) \end{aligned}$$

where

$$V(a) = \int_0^a y^{1-\epsilon} dF_a(y) \quad (7)$$

and  $dF_a(y) = f_a(y) dy$  is the differential of  $a$ 's CDF.

Let  $s_{X,i,j}$  and  $s_{I,i,j}$  be the market shares resulting from the revenues  $r_{X,i,j}$  and  $r_{I,i,j}$  earned by foreign firms from country  $i$  in country  $j$ . Then the market share of exporters relative to the market share of foreign affiliates is

$$\frac{s_{X,i,j}}{s_{I,i,j}} = \frac{\tau_{i,j}^{1-\epsilon} B_j (V(a_{X,i,j}) - V(a_{I,i,j}))}{B_j V(a_{I,i,j})} = \tau_{i,j}^{1-\epsilon} \left( \frac{V(a_{X,i,j})}{V(a_{I,i,j})} - 1 \right) \quad (8)$$

For symmetric countries the ratio is independent of  $i$  and  $j$ . It is increasing in  $a_X$  and decreasing in  $a_I$ .

## 4.3 Free entry condition

In equilibrium free entry will induce new firms to enter the market until the expected operating profit falls to zero. Therefore, the free entry condition requires that

$$\int_0^{a_{D,i}} (a^{1-\epsilon} B_i - f_D) dF_a(a) + \sum_{j \neq i} \left( \int_{a_{I,i,j}}^{a_{X,i,j}} ((\tau_{i,j} a)^{1-\epsilon} B_j - f_X) dF_a(a) + \int_0^{a_{I,i,j}} (a^{1-\epsilon} B_j - f_I) dF_a(a) \right) = f_E$$

where  $f_E$  is the one-time cost of market entry (see the working paper, footnote 17). This can alternatively be written as

$$\begin{aligned} &V(a_{D,i}) B_i + \sum_{j \neq i} (1 - \tau_{i,j}^{1-\epsilon}) V(a_{I,i,j}) B_j + \sum_{j \neq i} \tau_{i,j}^{1-\epsilon} V(a_{X,i,j}) B_j \\ &- (F_a(a_{D,i}) f_D + \sum_{j \neq i} F_a(a_{I,i,j}) (f_I - f_X) + \sum_{j \neq i} F_a(a_{X,i,j}) f_X) = f_E \end{aligned}$$

with  $V(a)$  defined in Eq. (7).

The first line represents expected profits, the second line expected costs (weighted by the probability of a firm being a domestic producer, exporter, etc.).

## 5 Parametrizing the productivity distribution

### 5.1 Distribution of productivity

Helpman et al. assume that productivity (not  $a!$ ) is Pareto-distributed with scale/location parameter  $b$  and shape parameter  $k$ . For a Pareto-distributed random variable (r.v.)  $x$  it must hold that  $x \geq b > 0$ , and  $x$  has a finite variance only if  $k > 2$ . Additionally, we need to assume that  $k > \epsilon - 1$  to ensure that the integral in  $V(a)$  converges. Helpman/Melitz/Yeaple require  $k > \epsilon + 1$ , which is a stronger assumption. We adopt this assumption from the start on, even though the reason for this will only become evident below.

The CDF and PDF of productivity are:

```
In[33]:= distAssump =  $\epsilon > 1 \ \&\& \ k > 2 \ \&\& \ b > 0 \ \&\& \ k > \epsilon + 1;$ 
```

```
In[34]:= TraditionalForm[prodcdf[p, b, k]]
```

Out[34]/TraditionalForm=

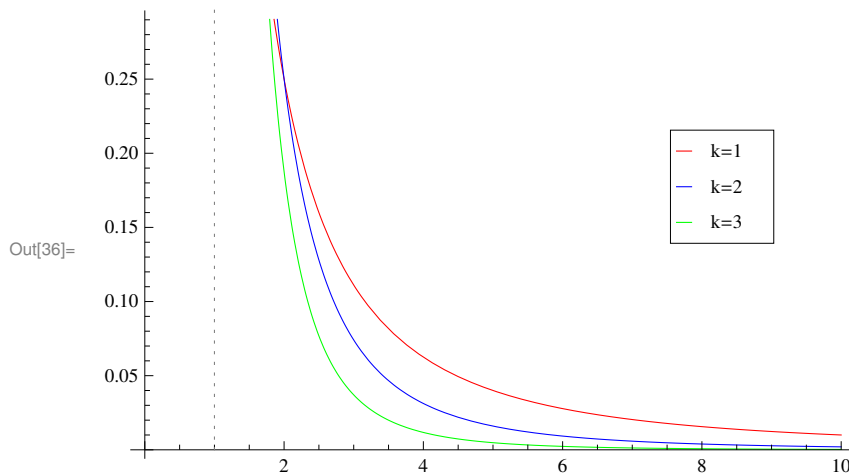
$$\begin{cases} 1 - \left(\frac{b}{p}\right)^k & p \geq b \\ 0 & \text{True} \end{cases}$$

```
In[35]:= TraditionalForm[prodpdf[p, b, k]]
```

Out[35]/TraditionalForm=

$$\begin{cases} k b^k p^{-k-1} & p \geq b > 0 \\ 0 & \text{True} \end{cases}$$

```
In[36]:= plotDiffK
```



The parameter  $k$  can be interpreted as a "natural" measure of productivity dispersion, with higher values of  $k$  leading to less variance in firm productivity (as shown in the figure above). The variance is defined as  $\frac{b^2 k}{(k-2)(k-1)^2}$  for  $k > 2$  and is decreasing in  $k$ :

```
In[37]:= Assuming[distAssump,
  FullSimplify[D[Simplify[Variance[ParetoDistribution[b, k]], k > 2], k] < 0]]
```

Out[37]= True

## 5.2 Distribution of $a$

To derive the distribution of  $a$ , the Pareto-distributed r.v.  $p \sim P(a_0, k)$  has to be transformed accordingly. As the labor input coefficient  $a$  is defined as  $a = 1/p$ , which is not an increasing transformation of  $p$ , we cannot use the transformation law to derive the distribution of  $a$ . Hence,

$$\begin{aligned} F_a(x) &= \mathbb{P}[a \leq x] \\ &= \mathbb{P}[1/p \leq x] \\ &= \mathbb{P}[p \geq 1/x] \\ &= 1 - \mathbb{P}[p \leq 1/x] \\ &= 1 - F_p(1/x) \end{aligned}$$

where  $F_a$  is the CDF of  $a$  and  $F_p$  is the CDF of  $p$ . The CDF and PDF of  $a$  can therefore be obtained as

$$\begin{aligned} F_a(x) &= 1 - F_p(1/x) \\ &= 1 - \left[ 1 - \left( \frac{b}{1/x} \right)^k \right] \\ &= (xb)^k \quad \forall 0 < x \leq 1/b \\ f_a(x) &= dF_a(x)/dx = kb^k x^{k-1} \quad \forall 0 < x \leq 1/b \end{aligned}$$

Note that the boundaries for the random variable  $a$  also have to be adjusted. As productivity  $p \geq b > 0$ , for the input coefficient  $a$  this implies that  $0 < a \leq 1/b$ .

The CDF and PDF of  $a$  obtained by transforming the productivity r.v. are:

```
In[38]:= TraditionalForm[acdf[a, b, k]]
```

```
Out[38]//TraditionalForm=
```

$$\begin{cases} (ab)^k & 0 < a \leq \frac{1}{b} \\ 1 & a > \frac{1}{b} \end{cases}$$

```
In[39]:= TraditionalForm[apdf[a, b, k]]
```

```
Out[39]//TraditionalForm=
```

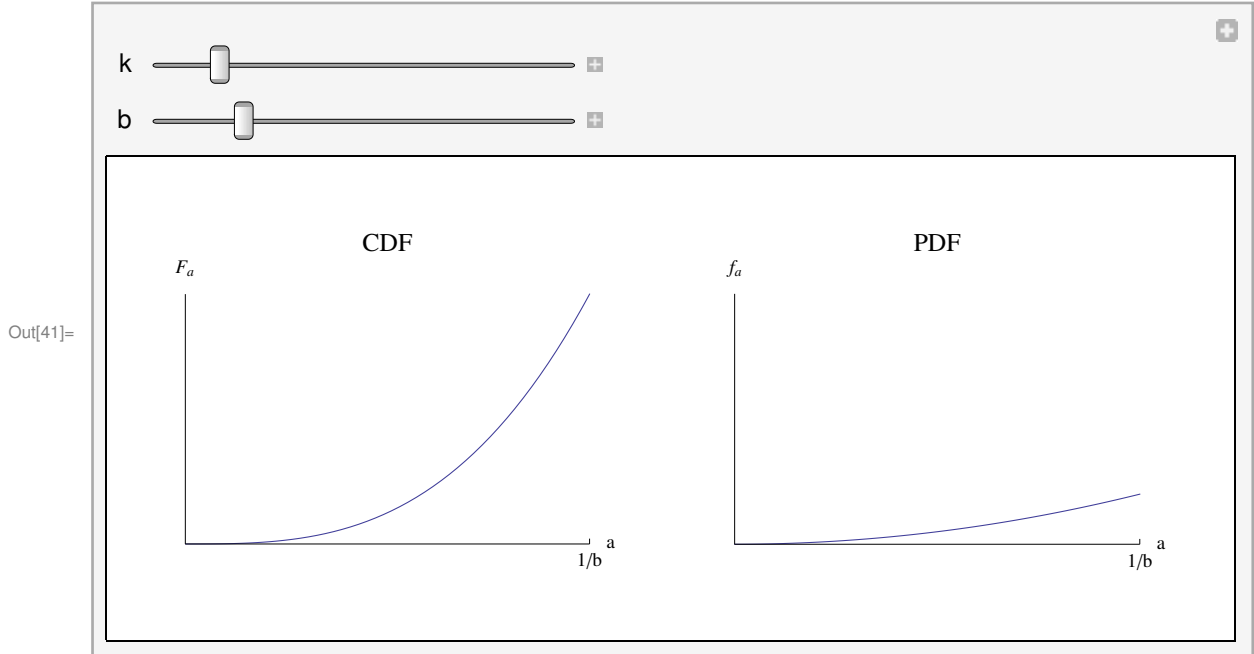
$$\begin{cases} k a^{k-1} b^k & a > 0 \wedge a b \leq 1 \\ 0 & \text{True} \end{cases}$$

To check whether the last expression can possibly be a PDF, integrate over its entire domain :

```
In[40]:= Integrate[apdf[a, b, k], {a, 0, \infty}, Assumptions -> distAssump]
```

```
Out[40]= 1
```

In[41]:= manipulateADist



### 5.3 Distribution of $V()$

There are two equivalent ways to determine the distribution of  $V(a)$ : either as a function of the r.v.  $p$  or as a function of the r.v.  $a$ .

In terms of productivity,  $V(a)$  can be redefined to be a function of  $p$  as

$$V(p) = \int_p^\infty x^{\epsilon-1} dF_p(x) \tag{9}$$

by setting  $a = 1/p$  where necessary and using the PDF of  $p$ . Evaluating the expression inside the integral, we get

$$x^{\epsilon-1} \left( \frac{k b^k}{x^{k+1}} \right) = \frac{k b^k}{x^{k-(\epsilon-1)+1}} \tag{10}$$

which is the PDF of a Pareto-distributed variable with location parameter  $b$  and shape parameter  $k - (\epsilon - 1)$ . Hence if productivity is Pareto-distributed, so is  $V()$ . As mentioned before, a Pareto-distributed r.v. has a finite variance only if its shape parameter is greater than 2. Therefore in this case we might want to assume that  $k - (\epsilon - 1) > 2 \Leftrightarrow k > \epsilon + 1$ , which is the restriction imposed by Helpman/Melitz/Yeaple.

The  $V(p)$  function evaluates to

In[42]:= vprod[p, b, k, ε] // TraditionalForm

Out[42]//TraditionalForm=

$$\begin{cases} \frac{k b^k p^{-k+\epsilon-1}}{k-\epsilon+1} & p \geq b \\ \text{Null} & \text{True} \end{cases}$$

Alternatively,  $V(a)$  can be defined as a function of  $a$  like Helpman/Melitz/Yeaple did in the paper (even though they never specified the distribution of  $a$ ):

$$V(a) = \int_0^a x^{1-\epsilon} dF_a(x) \tag{11}$$

The expression inside the integral evaluates to

$$x^{1-\epsilon}(k b^k x^{k-1}) = k b^k x^{k-\epsilon}$$

Integrating this, the resulting expression for  $V(a)$  is

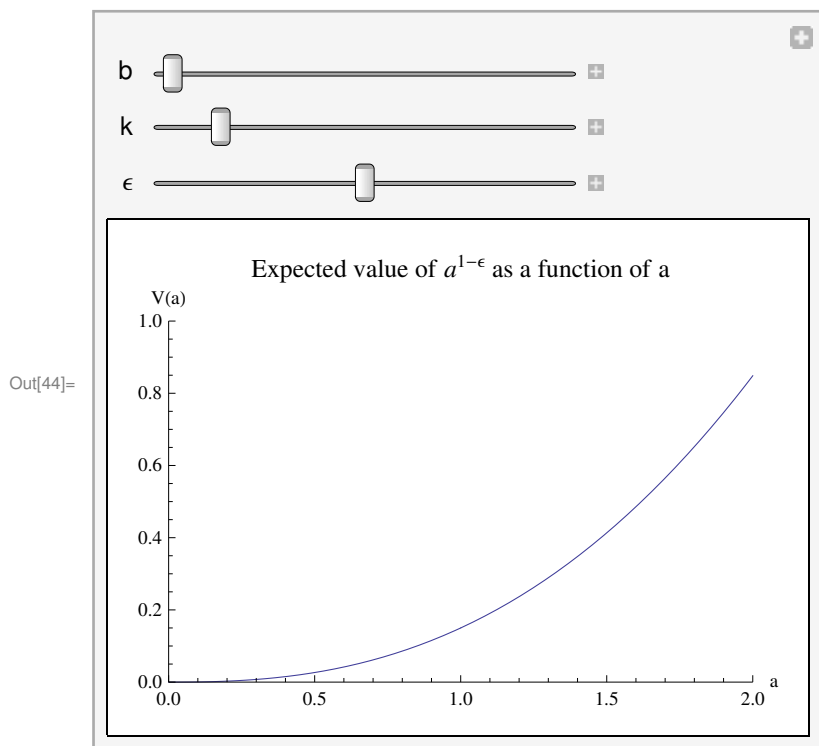
```
In[43]:= va[a, b, k, \epsilon] // TraditionalForm
```

Out[43]//TraditionalForm=

$$\frac{k b^k a^{k-\epsilon+1}}{k - \epsilon + 1}$$

This, of course, is equivalent to the expression derived above when setting  $p = 1/a$ . We see that  $V()$  is increasing in  $a$  and decreasing in  $p$ .

```
In[44]:= manipulateVa
```



### 5.4 Distribution of revenue $r$

Helpman/Melitz/Yeaple claim that  $V(a)$  reflects the distribution of revenue  $r$  up to a multiplicative constant. This can be verified as follows: revenue as a function of  $a$  and its inverse function are:

```
In[45]:= revenue[b, B, \epsilon] // TraditionalForm
```

Out[45]//TraditionalForm=

$$B b^{1-\epsilon}$$

```
In[46]:= invrevenue[r, B, \epsilon] // TraditionalForm
```

Out[46]//TraditionalForm=

$$\left(\frac{r}{B}\right)^{\frac{1}{1-\epsilon}}$$

(This holds for domestic and foreign affiliate sales and would have to be modified slightly for exports.)



The CDF and PDF of revenue can be obtained by transforming the r.v.  $a$  accordingly (revenue is a decreasing in  $a$ ).

```
In[47]:= revcdf[r, b, k, ε, b] // TraditionalForm
```

Out[47]//TraditionalForm=

$$\begin{cases} 1 - \left(b \left(\frac{r}{b}\right)^{\frac{1}{1-\epsilon}}\right)^k & \left(\frac{r}{b}\right)^{\frac{1}{1-\epsilon}} > 0 \wedge \left(\frac{r}{b}\right)^{\frac{1}{1-\epsilon}} - \frac{1}{b} \leq 0 \\ 0 & \left(\frac{r}{b}\right)^{\frac{1}{1-\epsilon}} - \frac{1}{b} > 0 \\ 1 & \text{True} \end{cases}$$

```
In[48]:= revpdf[r, b, k, ε, b] // TraditionalForm
```

Out[48]//TraditionalForm=

$$\begin{cases} \frac{k \left(b \left(\frac{r}{b}\right)^{\frac{1}{1-\epsilon}}\right)^k}{r(\epsilon-1)} & \left(\frac{r}{b}\right)^{\frac{1}{1-\epsilon}} > 0 \wedge \left(\frac{r}{b}\right)^{\frac{1}{1-\epsilon}} \leq \frac{1}{b} \\ 0 & \text{True} \end{cases}$$

Analogously to the  $V(a)$  function, the expected value of revenue for a firm with an input coefficient in  $(0, a)$  is:

```
In[49]:= revExp[a, b, k, ε, B] // TraditionalForm
```

Out[49]//TraditionalForm=

$$\begin{cases} \frac{B k b^k a^{k-\epsilon+1}}{k-\epsilon+1} & 0 < a \leq \frac{1}{b} \\ 0 & \text{True} \end{cases}$$

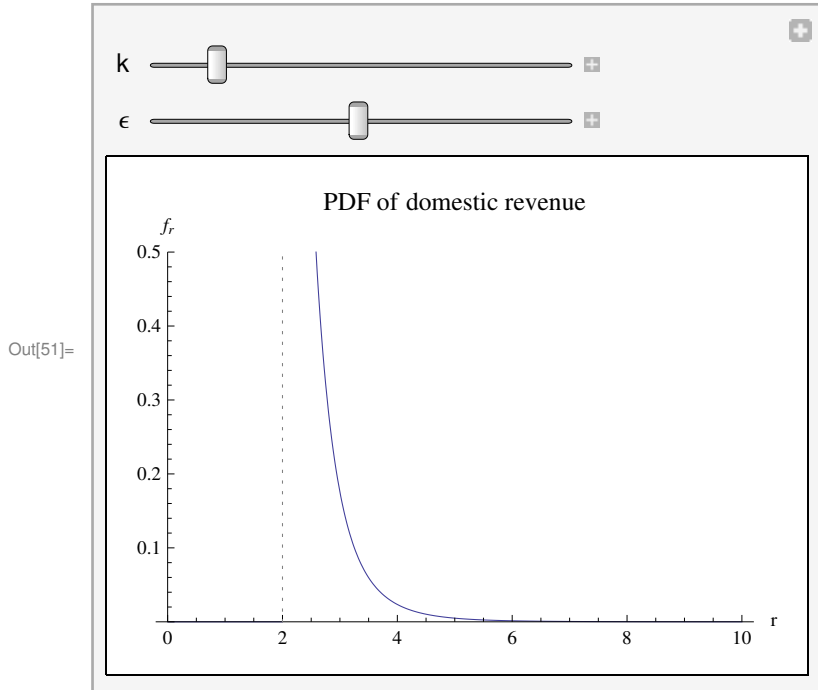
(Keep in mind that the boundaries have to be adjusted accordingly: an input coefficient  $a_i \in (0, a)$  corresponds to revenue  $r_i \in (r(a), \infty)$ , as revenue is decreasing in  $a$ .)

This is identical to the value of  $V(a)$  obtained above, but for a multiplicative constant:

```
In[50]:= FullSimplify[revExp[a, b, k, ε, B] / va[a, b, k, ε]]
```

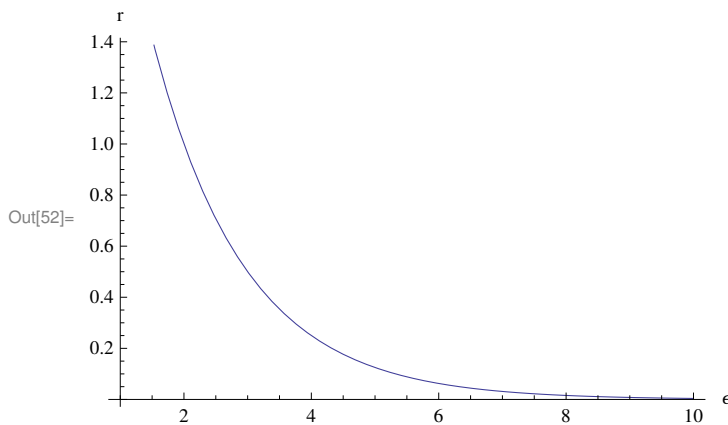
$$\text{Out[50]= } \begin{cases} B & a > 0 \ \&\& \ a \leq \frac{1}{b} \\ 0 & \text{True} \end{cases}$$

```
In[51]:= manipulateRevDist
```



From the interactive graph it can be seen that lower values of  $k$  (higher dispersion of productivity) lead to a higher dispersion in revenues. A higher  $\epsilon$  increases the dispersion of productivity and of  $V(a)$ , which also results in a more dispersed revenue (however,  $\epsilon$  also effects revenue via its role as elasticity of substitution and elasticity of demand, so there is an additional level effect on expected revenues).

```
In[52]:= plotRevElast
```



### 6 Exports vs. FDI (continued)

Returning to the discussion on relative market shares of exporters and foreign affiliates, from the expression for  $V(a)$  it is evident that

```
In[53]:= FullSimplify[va[a_x, b, k, epsilon] / va[a_i, b, k, epsilon]]
```

$$\text{Out[53]} = a_i^{-1-k+\epsilon} a_x^{1+k-\epsilon}$$

and hence the relative market share is

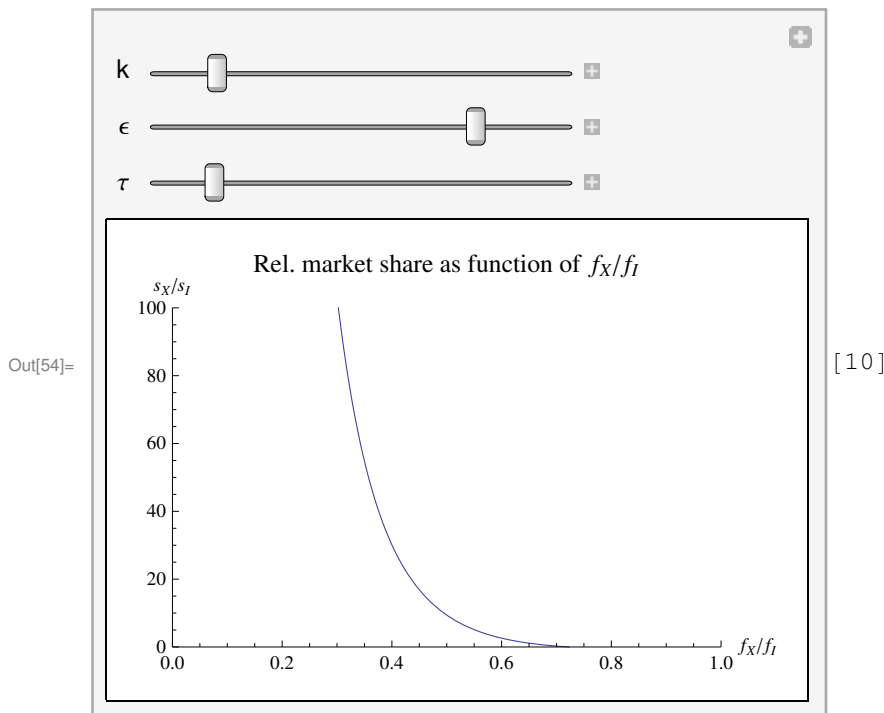
$$\frac{s_{X,i,j}}{s_{I,i,j}} = \tau^{1-\epsilon} \left( \left( \frac{a_X}{a_I} \right)^{k-(\epsilon-1)} - 1 \right) \tag{12}$$

Using Eq. (2) and Eq. (3) this can be expressed as

$$\frac{s_{X,i,j}}{s_{I,i,j}} = \tau^{1-\epsilon} \left( \left( \frac{f_I - f_X}{f_X} \frac{1}{\tau^{\epsilon-1} - 1} \right)^{\frac{k-(\epsilon-1)}{\epsilon-1}} - 1 \right) \tag{13}$$

For a given  $k$ ,  $\epsilon$  and  $\tau$ , the relative market share of exporters is decreasing in  $f_X/f_I$ . Higher relative fixed costs  $f_X/f_I$  induce more firms to choose market entry via FDI. Also, as Helpman/Melitz/Yeaple point out, the export share is increasing in  $k$  and decreasing in  $\epsilon$ . This effects is also observable in the interactive graph above. This is a central implication of the model which can be tested against empirical data: sectors with higher dispersion of domestic sales (be it due to higher dispersion of productivity or a higher elasticity of substitution) are expected to exhibit lower relative market shares of exporting activity abroad.

```
In[54]:= manipulateRelShare[10]
```



## 7 Special case: General equilibrium model with symmetric countries

*This section is still work in progress, as the solution to the equations presented in the working paper seems to yield a valid equilibrium (with  $a_D, a_X, a_I \in (0, 1/b)$ ) only for very restricted parameter values.*

(This section is based on the working paper.)

Additional assumptions which hold **within each sector** but can differ across sectors:

- Fixed costs  $f_E, f_D, f_X$  and  $f_I$  are equal in every country
- The distribution of productivity is identical across countries
- transport costs are the same for every pair of countries:  $\tau_{i,j} = \tau > 1$  for  $i \neq j$ .
- Countries can, however, **differ in size** (measured as size of the labor income,  $L_i$ ).

Given these assumptions, the cutoff levels of  $a$  will be identical for all countries:  $a_{D,i} = a_D, a_{X,i,j} = a_X$  and  $a_{I,i,j} = a_I$ . The free entry condition (Eq. (8)) for  $N$  countries simplifies to

$$V(a_D)B + (N-1)(1-\tau^{1-\epsilon})V(a_I)B + (N-1)\tau^{1-\epsilon}V(a_X)B \\ - (G(a_D)f_D + (N-1)G(a_I)(f_I - f_X) + (N-1)G(a_X)f_X) = f_E$$

with  $B_i = B$ . (For this to hold we have to assume that preferences are identical in each country as well, as  $B$  is determined by demand.)

For this special case a closed-form expression for  $B$  exists, however it is quite complex and does not yield any additional insights (also, *Mathematica* itself was unable to compute it, so this had to be done manually). See the accompanying mathematica package for the details. *Mathematica* can provide a numeric for  $B$  which can be compared to the analytical one:

```
In[55]:= eqn :=
  va[ad[B, ε, fD], b, k, ε] B + (n-1) (1 - τ1-ε) va[ai[B, ε, τ, fX, fI], b, k, ε] B +
  τ1-ε (n-1) va[ax[B, ε, τ, fX], b, k, ε] B -
  ((Simplify[acdf[x, b, k], 0 < x ≤ 1/b] /. x → ad[B, ε, fD]) fD +
  (n-1) (Simplify[acdf[x, b, k], 0 < x ≤ 1/b] /. x → ai[B, ε, τ, fX, fI])
  (fI - fX) +
  (n-1) (Simplify[acdf[x, b, k], 0 < x ≤ 1/b] /. x → ax[B, ε, τ, fX]) fX) ==
  fE
```

```
In[56]:= Bnumeric[b_, k_, ε_, τ_, n_, fD_, fX_, fI_, fE_] :=
  B /. FindRoot[eqn, {B, 1}]
```

```
In[57]:= Block[{fE = 1, fX = 2, τ = 1.1, ε = 1.5, fI = 3, fD = 1, b = 1, n = 2},
  TableForm[
    Table[{k, Bnumeric[b, k, ε, τ, n, fD, fX, fI, fE],
      Bcond[b, k, ε, τ, n, fD, fX, fI, fE]}, {k, 4, 10}],
    TableHeadings → {None, {"k", "Numerical", "Closed-form"}}]]
```

Bcond::noequil : No equilibrium exists for this parameter combination!

Bcond::noequil : No equilibrium exists for this parameter combination!

Bcond::noequil : No equilibrium exists for this parameter combination!

General::stop : Further output of Bcond::noequil will be suppressed during this calculation. >>

## References

- Dixit, Avinash K. and Stiglitz, Joseph E. (1977) : "Monopolistic Competition and Optimum Product Diversity". In : American Economic Review 67 (3), 297-308.
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- Helpman, Elhanan/ Melitz, Marc J. and Yeaple, Stephen R. (2004): "Export versus FDI with Heterogeneous Firms". In: American Economic Review 94(1), 300-316.